Preprocessing of temperature measurement by IR techniques using POD truncated basis for heat source estimation

QIRT 2014

7-11 July Bordeaux
The 12th International Conference on Quantitative InfraRed Thermography

7-11 July 2014
The 12th International Conference on Quantitative Infrared Thermography, Bordeaux

N. RANC[1], A. BLANCHE[2,4], D. RYCKELYNCK[3], A. CHRYSOCHOOS[2]

[1] PIMM (UMR CNRS 8006), Arts et Métiers ParisTech, 75013 Paris, France
[2] LMGC (UMR CNRS 5508), Université Montpellier 2, 34095 Montpellier, France
[3] Centre des matériaux (UMR CNRS 7633), Mines ParisTech, 91003, France
[4] UMET (UMR CNRS 8207), Université Lille 1, 59655 Villeneuve d'Ascq, France

Context of the study

Context of the study

Deformation of solids :

- temperature variation.
- heat sources : dissipation, thermomechanical coupling

Example : very high cycle fatigue loading
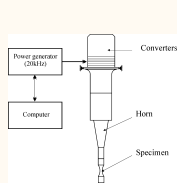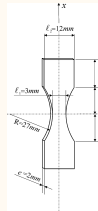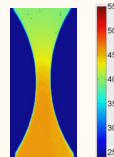

FIGURE : Ultrasonic machine

FIGURE : Specimen

FIGURE : Temperature field

Objectives :
To understand and model
the thermomechanical
behavior and the fatigue
damage evolution

Objective of the study

Heat source identification from temperature field measurement by IR thermography in order to understand the
thermomechanical behavior of the material

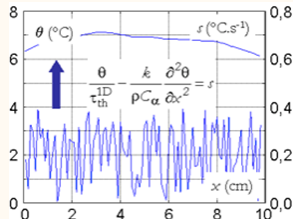## Problematic of heat source identification

Heat equation

$$\rho C_\alpha \left(\frac{\partial \vartheta}{\partial t} + \frac{\vartheta}{\tau_{th}^{1D}(x)} \right) - k \left(\frac{S'}{S} \frac{\partial \vartheta}{\partial x} + \frac{\partial^2 \vartheta}{\partial x^2} \right) = d_1(x,t) + s_{coupling}(x,t) = s(x,t), \quad (1)$$

where

- $s(x,t)$ longitudinal distribution of the mean heat source per cross section
- S, and S' section and its derivative
- $\tau_{th}^{1D}(x)$ being a time constant characterizing the heat losses

Difficulties

- Temperature are noisy
- Regularizing effect of heat equation

Method to reduce the noise

- Gaussian convolutive filtering : noise on the heat source, boundary effects
- Spectral method : projection onto eigenfunctions of the laplacian operator
- Local least squares fitting : local approximation of the temperature field

Objectives of the study

Objectives of this study

- to show that the projection on a reduced basis generated by proper orthogonal decomposition (POD) of thermal data improves the signal to noise ratio
- to underline the advantages of the use of a POD projection preprocessing before the classical methods : finite difference (DF), spectral solutions (SPS), local least square analytical approximation (LSQ)

Outline

1 Introduction
- Context of the study
- Problematic of heat source identification
- Objectives of the study

2 Application of POD preprocessing to thermal data

3 Benchmark test

4 Comparative analysis

5 Conclusion

Outline

Application of
POD
preprocessing

POD Preprocessing
method

Example of POD basis
construction

POD Preprocessing method

Introduction

Application of POD preprocessing

POD Preprocessing method

Example of POD basis construction

Benchmark test

Comparative analysis

Conclusion

The reduced basis representation of the temperature field

$$\vartheta(x, t) = \sum_{k=1}^{N} a_k(t)\varphi_k(x), \qquad (2)$$

where $(\varphi_k)_{k=1,\ldots,N}$ are the proper orthogonal modes (POMs) of the POD reduced basis.

POD basis construction

The reduced basis is related to thermal field obtained from solutions of chosen heat diffusion problem $(\vartheta^{(r)})_{r=1,\ldots M}$:

$$\varphi_k = \arg\max_{\psi} \frac{\sum_{r=1}^{M} \int_0^{t_f} \left(\int_\Omega \vartheta^{(r)}(x, t) \, \psi(x) \, d\Omega \right)^2 \, dt}{\int_\Omega \psi^2(x) \, d\Omega}, \qquad (3)$$

where Ω is the spatial domain and $[0, t_f]$ is the time interval.

Eigenvalue related to the POM φ_k

$$\sigma_k^2 = \frac{\sum_{r=1}^{M} \int_0^{t_f} \left(\int_\Omega \vartheta^{(r)}(x, t)\varphi_k(x) \, d\Omega \right)^2 \, dt}{\int_\Omega \varphi_k^2(x) \, d\Omega} \qquad (4)$$

σ_k^2 represents the energy of mode φ_k

Example of POD basis construction

Diffusion problem

- Choice of putative heat source $(s^{(r)})_{r=1,\dots M}$: time constant point sources
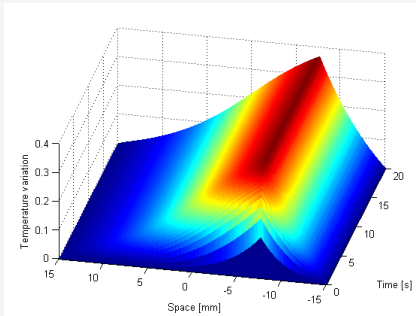- Temperature field calculation $\vartheta^{(r)}(x,t)$

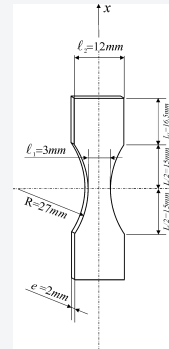FIGURE : Calculated temperature field for a point source at $x = -7.5$ mm

FIGURE : Specimen geometry.

$$\rho C_\alpha \left(\frac{\partial \vartheta^{(r)}}{\partial t} + \frac{\vartheta^{(r)}(x)}{\tau_{th}^{1D}(x)} \right) - k \left(\frac{S'}{S} \frac{\partial \vartheta^{(r)}}{\partial x} + \frac{\partial^2 \vartheta^{(r)}}{\partial x^2} \right) = s^{(r)}(x), \quad \vartheta^{(r)}(\pm \frac{L}{2}) = 0. \quad (5)$$

Example of POD basis construction

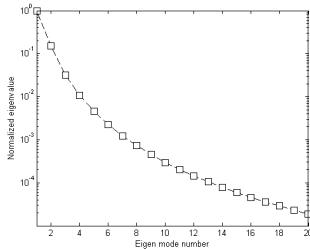## First eigenvalues and proper orthogonal modes

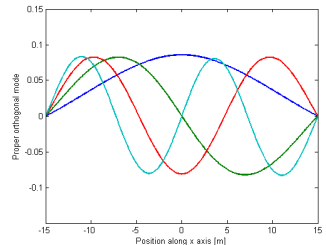FIGURE : Normalized distribution of eigenvalues

FIGURE : First proper orthogonal modes

- the eigenvalues represents the level of energy of each mode and decrease quickly ;
- It is thus possible to truncated this POD base without affecting the total energy of the system ($N_m \approx 10$ in our case).

Outline

QIRT 2014
Bordeaux

Reference heat source and associated temperature

A benchmark test with a particular heat source is chosen :

- to test the robustness of the POD method ;
- to show advantages of the use of POD preprocessing before classical identification method.
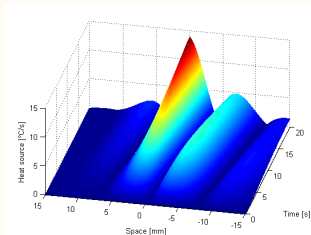
Reference heat source

FIGURE : Reference heat source

- highly heterogeneous field ;
- slowly time evolution.
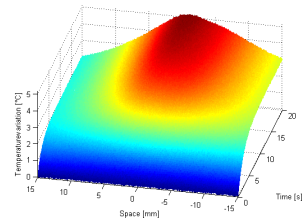
Associated noisy temperature

FIGURE : Associated noisy temperature

- dissymmetric Dirichlet boundary conditions ;
- Gaussian white noise with a three standard deviation of 50 mK.

QIRT 2014
Bordeaux

Projection on a truncated POD basis

The temperature field is filtered by projection on a truncated POD basis :

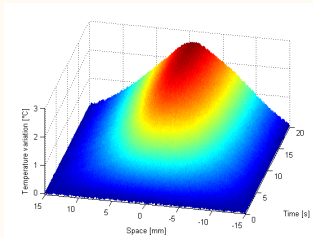#### Boundary conditions modification

FIGURE : Boundary conditions modification

- superimposing the solution of the thermal problem with a null source and boundary conditions equal to the experimental value
- zero temperature variation on the boundary conditions

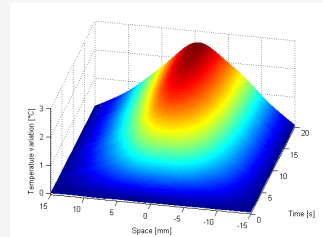#### Temperature field after projection

FIGURE : Temperature field after projection

- projection on the truncated POD basis of 10 modes

Heat source calculation

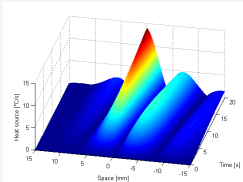## Heat source calculation by the finite difference method :
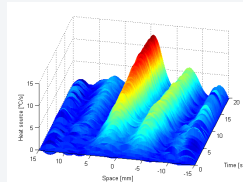

FIGURE : Reference heat source

FIGURE : Temperature field after projection
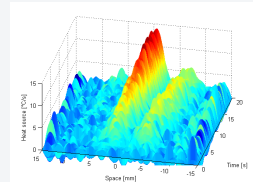
FIGURE : Temperature field after projection

- projection on the truncated POD basis of 10 modes

- projection on the truncated POD basis of 15 modes

Remarks :

- POD projection and finite difference method allow to correctly capture inhomogeneities and time evolution ;
- The increase of the number of mode improve the maximum heat source but increase the noise particularly in the region where the heat sources are small ;
- the error on the estimate of the maximum heat source is about $0.83 \, °C/s$

QIRT 2014
Bordeaux

Error analysis

Introduction

Application of
POD
preprocessing

Benchmark test

Reference heat source
and associated
temperature

Projection on a
truncated POD basis

Heat source calculation

Error analysis

Comparative
analysis

Conclusion

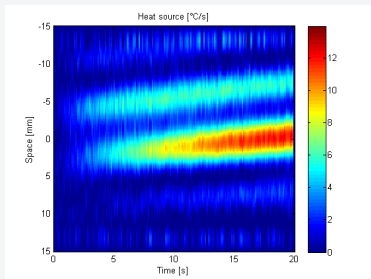Error on the heat source estimate :
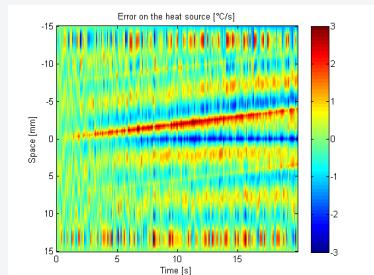

FIGURE : Identifed heat source

FIGURE : Difference between the estimate and the reference heat
sources.

- increase of the error in the zone of high gradient
- increase the noise in the region where the heat source remain small.

QIRT 2014
Bordeaux

Outline

Introduction

Application of
POD
preprocessing

Benchmark test

Comparative
analysis

Conclusion

QIRT 2014
Bordeaux

Comparative analysis

Introduction

Application of
POD
preprocessing

Benchmark test

**Comparative
analysis**

Conclusion

method	mean $(\text{x } 10^{-3}\ ^\circ C/s)$	std $(\text{x } 10^{-3}\ ^\circ C/s)$	parameter	CPU time (s)
FD	-	-	-	-
SPS	-11	726	$N_{ef} = 10$	37.95
LSQ	21	429	$N_x = 15\ N_t = 15$	5.37
POD+FD	1	750	$N_m = 10$	0.52+0.33
POD+SPS	-12	734	$N_m = 15\ N_{ef} = 10$	0.61+11.64
POD+LSQ	-9	383	$N_m = 15\ N_x = 10\ N_t = 10$	0.61+5.34

TABLE : Heat source assessments using noisy thermal data.

Comparative analysis of the use of POD method before heat source assesment :

- FD method :
  - Without POD preprocessing, the FD method gives too noisy estimates of heat sources because the 2nd order derivative of noisy data considerably amplifies the noise ;
  - heat sources can be now calculated with the FD method after POD Preprocessing.

- SPS method
  - the quality of the results is similar with and without POD preprocessing
  - this method is really time consuming ; POD preprocessing allows a more rapid convergence of the projections for the same number of eigenfunctions.

- LSQ method
  - POD preprocessing slightly improves the standard deviation.
  - The main advantage of this technique is that it allows the reduction of the approximation zone.

Outline

Introduction

Application of POD preprocessing

Benchmark test

Comparative analysis

Conclusion

Conclusion

Conclusions

- main objective of the presentation : that the use of a truncated POD base of thermal data improves significantly the heat source assessments ;
- a 1D benchmark .
- the general advantages of POD preprocessing :
    - the POM are computed once the diffusion problem is defined. They are computed once and for all ;
    - the number of POM(s) to be considered in the preprocessing depends on the noise characteristic but generally remains small ;
    - the CPU time necessary to project the thermal data are almost negligible compared with those necessary to compute the heat sources ;
    - the POD preprocessing allows to use the FD method with noisy thermal data to calculate heat sources ;
    - noise reduction by POD enables us to consider a greater number of eigenfunctions for the spectral method and to reduce the fitting zone in the least-square method in order to limit the crushing of thermal gradient for example.

Thanks for your attention

Any questions ?

Nicolas Ranc **Associate Professor**

Arts & Métiers ParisTech - Centre de Paris
Laboratoire PIMM
151 boulevard de l'Hopital, 75013 Paris, France
nicolas.ranc@ensam.eu